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A Projective Similarity/Eddy-Viscosity Model for Large-Eddy Simulation

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1 Introduction

Since most turbulent flows possess far more eddies than can be computed from the Navier-Stokes equations, an approximate description of the dynamics of the 'large-eddies' is sought in which the 'small-eddies' need not be computed explicitly. In large-eddy simulation (LES), this is achieved by applying a frequency low-pass filter to the equations; see e.g. [1]. The interaction between the large (filtered) and small (residual) eddies is then represented by the commutator of the filter and the nonlinear term in the Navier-Stokes equations. Hence, this commutator has to be modeled in terms of the filtered velocity to obtain the intended, approximate, large-eddy dynamics. For this, one usually resorts to an eddy-viscosity model cf. [2]-[3], a (scale) similarity model cf. [4]-[5], or a mix thereof cf. [6]. Similarity models have the proper mathematical structure. Additionally, they correlate well with the real commutator. Yet, their leading term has directions of negative diffusion [7]-[8]. In this paper, we propose to stabilize similarity models by projecting them onto an eddy-viscosity model. The projection eliminates the dynamically unstable part and results in a self-calibrating eddy-viscosity. In comparison with mixed models, we do not add a dissipative term to stabilize the similarity model, but instead we remove the dynamically unstable art. The resulting projective similarity/eddy-viscosity model is successfully tested for a turbulent channel flow at $\mathrm{Re}_\tau = 2520$ (based on the friction velocity and channel half-width).

2 Spatial Filter

We consider the elliptic, differential filter [9]

$$\bar{u} = \mathcal{F}u = u + \alpha^2 \nabla^2 u, \tag{1}$$

where α parameterizes the filter-length. The boundary conditions that supplement the Navier-Stokes equations are applied to the filter too. This filter is generic in the sense that any symmetric convolution filter can be approximated by (1), where the error is of the order α^4. Additionally, it has been shown in [10] that the approximate inverse of (1),

$$u \approx \tilde{u} = \tilde{\mathcal{F}}^{-1}\bar{u} = (1 - \alpha^2\nabla^2)\bar{u}, \tag{2}$$

forms the essence of the recently proposed 'alpha-model'.

The commutator of the filter (1) and the Navier-Stokes operator is given by $\nabla \cdot \tau$, where the subfilter stress τ depends upon the velocity-gradient:

$$\tau_{ij}(\nabla u) = 2\alpha^2 \nabla u_i \cdot \nabla u_j - \alpha^4 \nabla^2 u_i \nabla^2 u_j. \tag{3}$$

3 Projective Similarity/Eddy-Viscosity Model

Similarity models are based upon an approximate defiltering procedure. With the help of the approximate defilter given by (2), we can model the subfilter stress τ_{ij} by replacing the velocity u in (3) by the right-hand side of (2). The resulting similarity model $\tau_{ij}(\nabla u) \approx \tau_{ij}(\nabla \tilde{\mathcal{F}}^{-1}\bar{u})$ possesses the correct mathematical structure; particularly, it satisfies all properties of a commutator. Additionally, the correlation between the approximation $\tau_{ij}(\nabla \tilde{u})$ and $\tau_{ij}(\nabla u)$ is generally strong, typically between 0.6 and 0.9. Yet, this model is not unconditionally stable as the leading term of $\tau_{ij}(\nabla \tilde{u})$ has directions of negative dissipation. Therefore, we propose to remove the dynamically unstable part of $\tau_{ij}(\nabla \tilde{u})$ by means of a projection onto an eddy-viscosity model of the form

$$-\tau_{ij}(\nabla u) + \tfrac{1}{3}\delta_{ij}\tau_{kk}(\nabla u) \approx \nu(\partial_j \bar{u}_i + \partial_i \bar{u}_j), \tag{4}$$

where the isotropic part $\frac{1}{3}\delta_{ij}\tau_{kk}(\nabla u)$ need not be modeled, as it can be incorporated into the pressure. The projection results into a self-calibrating eddy-viscosity $\nu(x, t)$ which is computed such that the best approximation of $-\tau_{ij}(\nabla \tilde{u}) + \frac{1}{3}\delta_{ij}\tau_{kk}(\nabla \tilde{u})$ is obtained in least-square sense,

$$\min \int e_{ij} e_{ij} \mathrm{d}V, \tag{5}$$

where integral extends over the entire flow domain, the residuals are $e_{ij} = \tau_{ij}(\nabla \tilde{u}) - \frac{1}{3}\delta_{ij}\tau_{kk}(\nabla \tilde{u}) - \nu(\partial_j \bar{u}_i + \partial_i \bar{u}_j)$ and the minimum is computed (with respect to ν) subject to the stability constraint $\nu + 1/\mathrm{Re} > 0$. The solution of this constrained variational problem reads

$$\nu = \frac{(\tau_{ij}(\nabla \tilde{u}) - \frac{1}{3}\delta_{ij}\tau_{kk}(\nabla \tilde{u}))(\partial_j \bar{u}_i + \partial_i \bar{u}_j)}{(\partial_n \bar{u}_m + \partial_m \bar{u}_n)^2}, \tag{6}$$

if the right-hand side is larger than $-1/\mathrm{Re}$; and $\nu = -1/\mathrm{Re}$ otherwise.

In summary, the projective similarity/eddy-viscosity model is given by (4), where the eddy-viscosity ν is computed according to (6), with $\tilde{u}$ as in (2) and τ_{ij} as in (3).

4 An a Posteriori Test: Turbulent Channel Flow

As a first step in application of the proposed model, it is tested for a turbulent channel flow at $\mathrm{Re}_\tau = 2520$ by comparing the results with those of Comte-Bellots wind tunnel experiment [11]. Besides we will compare with experiments by Wei and Willmarth [12], which were performed at lower Re_τ (1025-1650), and since making good near-wall measurements is difficult, we also make a comparison with a direct numerical simulation (DNS) at $\mathrm{Re}_\tau = 590$ [13]. Obviously, the comparison with DNS is to be restricted to the direct vicinity of the wall, where Reynolds-number effects can be properly scaled.

As usual, the flow is assumed to be periodic in the stream- and spanwise direction. The computational grid consists of 128 streamwise points, 64 spanwise points and 300 points between the channel walls. All LES-results are approximately defiltered by means of (2) in order to compare them directly with the available experimental data. Details of the computational procedure are discussed elsewhere [14]; here we focus on the principal results.

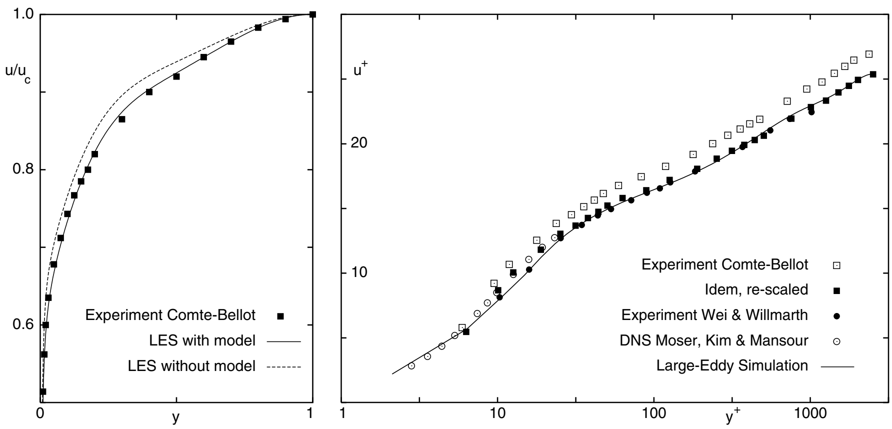

Fig. 1. Mean streamwise velocity. In the left-hand side the velocity is normalized by the centre-line velocity. The right-hand side figure shows u^+ as function of y^+.

The least to be expected from a LES is a good prediction of the mean flow. As can be seen in Fig. 1 (left) the present LES satisfies that minimal requirement: without any models the prediction of the mean flow worsens significantly. With model, the agreement with the data of Comte-Bellot is good. Yet, the friction velocities u_τ differ. Comte-Bellot deduced $u_\tau = 0.0416$. We have $u_\tau = 0.0442$, which is in good agreement with Dean's result $u_\tau = 0.0445$. Therefore, we have rescaled the Comte-Bellots mean-velocity profile with the help of our u_τ. After this rescaling, the result of Comte-Bellot shows an excellent agreement with the data in [12]-[13] and with the present LES.

As can be seen in Fig. 2, the turbulent intensities agree well, except for the spanwise fluctuations, which agree fairly: the spanwise turbulence intensity of

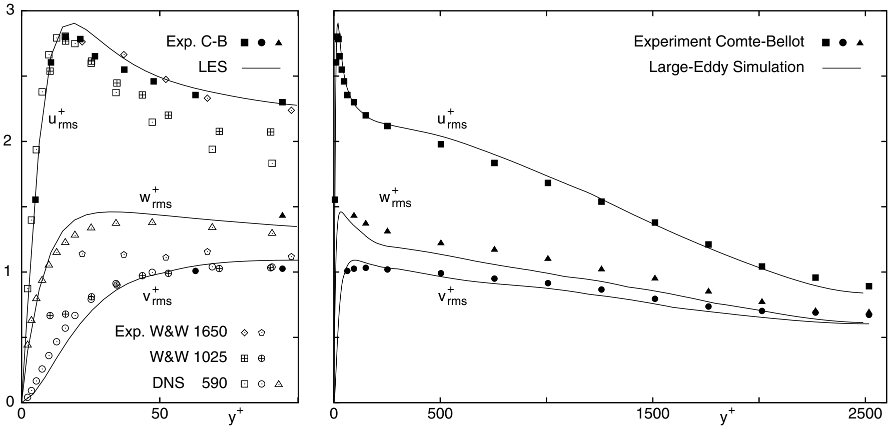

Fig. 2. The root-mean-square of the fluctuating velocity at $\mathrm{Re}_\tau = 2520$. Here, the results by Comte-Bellot are rescaled, like in Fig. 1. Near the wall (left-hand figure) two experiments by Wei and Willmarth [12] are shown, namely at $\mathrm{Re}_\tau = 1650$ and $\mathrm{Re}_\tau = 1025$. The DNS [13] has been performed at $\mathrm{Re}_\tau = 590$. Note: for $y^+ > 30$ the comparison with low-Re_τ data does not hold due to Reynolds-number effects.

Comte-Bellot is consistently higher than that of the LES. Near the wall there also exists a good agreement between the streamwise intensity measured by Wei and Willmarth ($\mathrm{Re}_\tau = 1650$) and the present result. In summary, good agreement with previously reported experimental results is observed.

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